Efficient Implementation of FIR Type Time Domain Equalizers for MIMO Wireless Channels via M-LESQ

Yang LIU
Information Security National Engineering Laboratory
School of Information Security Engineering, Shanghai Jiao Tong University
Shanghai, China
liu-yang@sjtu.edu.cn

Ping YI[1,2]
[1]Information Security National Engineering Laboratory, School of Information Security Engineering, Shanghai Jiao Tong University,Shanghai, China
[2]Key Laboratory of Child Development and Learning Science of Ministry of Education, Southeast University,Nanjing, China
yiping@sjtu.edu.cn

Yue WU
Information Security National Engineering Laboratory
School of Information Security Engineering, Shanghai Jiao Tong University
Shanghai, China
wuyue@sjtu.edu.cn

Abstract—**Efficient implementation of FIR (finite-impulse-response) equalizers for wireless channels has attracted much attention these days as equalizations can improve the link performance in hostile mobile radio environment by compensating for inter-symbol interference created by multipath within time dispersive channels. Meanwhile, the MIMO (multiple-input-multiple-output) technique becomes a trend in wireless channel design. On the other hand, we have observed a Least-Squares rational function system identification algorithm, which approximates FIR impulse response with IIR (infinite-impulse-response) structures effectively. In this paper, we exploit the approximation algorithm and extend the algorithm for a MIMO response approximation which can generate a hardware-efficient MIMO IIR structure for FIR type time domain equalizers in the wireless system. We demonstrate the efficiency and accuracy of our method with MIMO modeling examples**.

Keywords- Approximation methods, FIR digital filters, IIR digital filters, MIMO systems, Least squares methods, Time domain equalizers

I. INTRODUCTION

There is a fast increasing demand for high-requirement of data rates communication; meanwhile, the lack of wireless spectrum has drawn the consideration in recent years of MIMO wireless communication systems that can support much higher data rates than traditional single-input-single-output (SISO) wireless channels. Then the multiple-input-multiple-output (MIMO) wireless channels are applied.

However, MIMO also put a huge impact on wireless communication due to the vector nature of the channel precludes the use of trellis-based equalization techniques because of the excessively large number of states required, thus it's no doubt that the task of channel equalization becomes a challenging issue, and at the same time the MIMO decision-feedback equalizers have a high computational complexity.

As a result, many MIMO wireless systems emphasize on the equalization by purposefully trading off some amount of system performance against a lower implementation complexity. In [1]'s work, research has introduced the time domain equalizer which can be employed to design an FIR type MIMO equalizer that equalizes exactly the channel at the receiver.

On the other hand, in digital filter design, some researches have put their focus on the infinite-impulse-response (IIR) approximation of finite-impulse-response (FIR) types, e.g., [2]–[7]. This is motivated by

1). IIR design methodology through matching to a prescribed FIR prototype;

2). Hardware savings because of the fewer multipliers in IIR structures.

SISO (single-input-single-output) Least-Squares (LS) [4] is shown to be accurate and effective in approximation of FIR type responses with IIR structures, comparing to other algorithms [6]. It exploits the advantages of simple calculation, and avoids state-space realization, eigenvalue computation or initial pole assignment, and thus shuns from the problem of the numerical-sensitive calculation. Its elegant framework has been extended to the interconnected macro-modeling [9]. So far, its MIMO modeling configuration have not been developed and applied for the MIMO wireless channel design process.

In this paper, we look into the SISO Least-Squares algorithm, and propose M-LESQ (MIMO LEast SQuares) for practical designing and implementation in FIR equalizers in MIMO wireless channel implementation. The FIR equalizer design is introduced in Section 2. The SISO LS algorithm brief derivation, its implementation discussion, the algorithm efficiency analysis and the MIMO extension are shown in Section 3. Numerical examples of practical equalizer design in Section 4 confirm the remarkable efficiency and accuracy of the algorithm, which generates equalizers with a smaller hardware cost and a robust implementation.

Project supported by Nation Natural Science Foundation of China (Grant No. 60803117), the National High Technology Development 863 Program of China (Grant No. 2009AA01Z118, No.2006AA01Z436, No. 2007AA01Z452) Shanghai Municipal Natural Science Foundation (Grant No.09ZR1414900), and by Intel College Student Innovation Program (Grant No. Intel 091024801)

II. MIMO WIRELESS SYSTEM AND FIR TIME DOMAIN EQUALIZER

A. MIMO Wireless System

The promising observation of increased channel capacity, reliability, and range of multiple-input multiple- output (MIMO) systems makes multiple antenna systems received a significant amount of interest by the research community in recent years. MIMO wireless systems obtain large diversity and capacity gains by employing multi-element antenna arrays at both the transmitter and receiver. Achieving the gains of the MIMO channel requires coding that can take advantage of these channel resources.

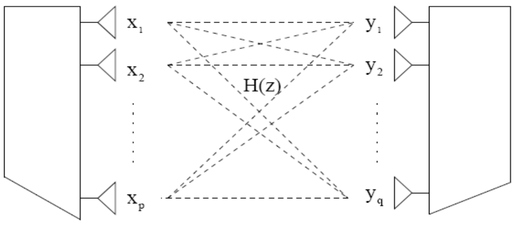

Figure. 1 Multiple transmit and receive antennas communication system.

In our paper, let's consider a MIMO channel model has p transmit and q receive antennas,

$$y = Hx + v, \quad (1)$$

where $x \in C^{p\times 1}$ is the input vector, $y \in C^{q\times 1}$ is the output vector, $v \in C^{q\times 1}$ is an additive zero mean circularly symmetric complex Gaussian noise vector distributed as $N(0,\sigma^2 I)$, and $H \in C^{q\times p}$ is a random channel matrix, where each element is an independent circularly symmetric complex Gaussian random variable with variance one.

We can then design MIMO FIR equalizers that will be applied to the channel blocks.

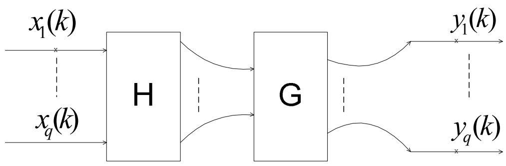

Figure.2 MIMO communication system with equalizer.

When the number p of transmit antennas is larger than the number q of receive antennas, an $p \times q$ matrix FIR equalization $G(z) = \sum_{k=0}^{Ord} G(k) z^{-k}$ with order Ord is used to equalize the channel at the receiver. The received sampled signal that passed through an $p \times q$ matrix FIR equalization filter $G(z) = \sum_{k=0}^{Ord} G(k) z^{-k}$ can be written as

$$y(n) = H(n) * x(n) * G(n) + v(n) * G(n), \quad (2)$$

Similarly, when q > p, the received sampled signal that passed through an $p \times q$ matrix FIR equalization filter $G'(z) = \sum_{k=0}^{Ord} G'(k) z^{-k}$ can be written as

$$y'(n) = H(n) * x(n) * G'(n) + v(n) * G'(n), \quad (3)$$

B. Time Domain Equalizer Overview

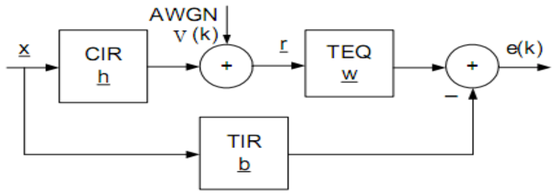

Figure. 3 Block Diagram of TEQ.

As illustrated in Fig.3, a receiver equalization scheme comprising a time domain equalizer (TEQ) is proposed these years. Many research investigations have been undertaken to define an effective TEQ initialization algorithm.

As we can seen in Fig.3, the received data is given by,

$$y(k) = h^T \cdot x' + v(k), \quad (4)$$

Hence the error signal after the TEQ is given as

$$e(k) = w^T \cdot r - b^T \cdot x, \quad (5)$$

The squared error is given by,

$$E\{|e(k)|^2\} = w^T \cdot R_{rr} \cdot w^* + b^T \cdot R_{xx} \cdot b^* - w^T \cdot R_{rx} \cdot b^* - b^T \cdot R_{rx} \cdot w^*, \quad (6)$$

The optimal equalizer tap coefficients can be obtained by solving for the Minimum Mean Squared Error (MMSE).

The TEQ coefficients w can be obtained iteratively through steepest gradient methods such as Least Mean Square (LMS),

$$w^{k+1} = w^k - \Delta' \cdot e(k) \cdot r^*, \quad (7)$$

where Δ' is the LMS convergence control parameter. However, imposing an exact equalizer structure is still too constraining in some occasions, so the efficient implementation of this FIR equalizers becomes a significant issue in the MIMO communication system.

III. EFFICIENT IMPLEMENTATION OF EQUALIZER

The idea of IIR approximation is to approximate a FIR response $G(z) = \sum_{n=0}^{L} h_n z^{-n}$ using an Mth order IIR structure $F(z) = P(z)/Q(z) = \sum_{n=0}^{M} \left(p_n z^{-n} / q_n z^{-n} \right)$. Different from and advantageous over other related existing IIR approximation algorithms, a promising observation in LS [4] is the use of Walsh's interpolation theorem [7] and complementary signal [8]. Based on these findings, a numerical-robust and efficient algorithm is developed for IIR approximation. In this section, the SISO numerator and denominator calculation are first discussed, and then the MIMO FIR system (i.e. equalizer responses) approximation using a common-denominator IIR structure is proposed in the later subsection.

A. Determination of Numerator

IIR approximation requires non-linear computations. But if the prescribed poles $\{\alpha_1,\alpha_2,\ldots,\alpha_M\}$ are given, by the Walsh's Theorem [7], the best approximation in the Least-Squares sense to $G(z)$ is the unique function that interpolates to $G(z)$ in $z=\{\infty,1/\alpha_1^*,\ldots,1/\alpha_M^*\}$, where * denotes complex conjugate. Using this theorem, we can separate the approximation problem into the determination of the numerator and denominator coefficients. Taking advantage of Walsh's Theorem, the numerator can be calculated through an interpolation problem of $P(z)/Q(z)$ at the points z_k in z, which means

$$G(z)\big|_{z=z_k}=\left(P(z)/Q(z)\right)\big|_{z=z_k}, \quad (8)$$

where $k=0,1,\ldots,M$, assuming there are no repeated poles in $\{\alpha_1,\alpha_2,\ldots,\alpha_M\}$. The interpolation problem forms M+1 equations, which are linear in the unknowns $p_n(n=0,1,\ldots\ldots,M)$. Here we define the error term:

$$\Delta(z)=G(z)-\left(\mathrm{P}(z)/Q(z)\right), \quad (9)$$

Since the approximant is the impulse response approximation of a system, the interpolation condition (8) can be used to describe $\Delta(z)$ by a cascade of a FIR filter *R(z)* and an all-pass filter *A(z)*,

$$\Delta(z)=A(z)R(z)=\frac{z^{-M}Q(z^{-1})}{Q(z)}R(z), \quad (10)$$

where $R(z)=\sum_{k=0}^{L-1}r_kz^{-k}$. With (9), the interpolation problem can be described as an input-output description of a digital filter operation,

$$r_{L-1-n}=u_n \text{ for } n=0,1,\ldots,L-1, \quad (11)$$

$$P(z)=G(z)Q(z)-z^{-(M+1)}Q(z^{-1})R(z), \quad (12)$$

where $U(z)=z^{-L}G(z^{-1})A(z)=\sum_{k=0}^{L-1}u_kz^{-k}$.

In summary, *P(z)* can be determined from *G(z)* and *Q(z)*. Thus, we only consider *Q(z)* in the overall approximation process and try to find a way to minimize $\|\Delta(z)\|$.

B. Determination of Denominator

To calculate the location of poles, instead of using eigenvalue calculation or root finding techniques in conventional methods, LS determines *P(z)* through a "energy-conserved" all-pass filter characteristic. The idea of complementary signal is that if the time-reversed response signal $z^{-n}G(z^{-1})=h(-n)$ is fed into the all-pass filter, then the energy of the response $a(n)=A\left[h(-n)\right]$ can be distributed as

$$\sum_{n=-\infty}^{\infty}\left|a[n]\right|^2=\sum_{n=-L}^{0}\left|a[n]\right|^2+\sum_{n=1}^{\infty}\left|a[n]\right|^2, \quad (13)$$

where the first summation is the approximation error energy and the second summation is the energy of the approximant. The algorithm objective is to design an all-pass filter which minimizes the approximation error energy. In numerical calculation, (10) is modified for designing an all-pass operator $A^{(k)}(z)$ of a given order M with

$$\Delta^{(k)}(z)=A^{(k)}(z)z^{-1}R^{(k)}(z)=\frac{z^{-M}Q^{(k)}(z^{-1})}{Q^{(k-1)}(z)}z^{(-1)}R^{(k)}(z), \quad (14)$$

where k is the number of iterations. All-pass operator (10) converges to an all-pass function, the optimal denominator polynomial *Q(z)* is found. Since $\|\Delta^{(k)}(z)\|=\|R^{(k)}(z)\|=\|U^{(k)}(z)\|$ in (10) and (11), LS involves a digital filtering operation and a set of over-determined equations to minimize $\|\Delta^{(k)}(z)\|^2$.

To construct the least-squares problem, first we define

$$X^{(k)}(z)=z^{-L}G(z^{-1})/Q^{(k-1)}(z)=\sum_{n=0}^{\infty}x^{(k)}(n)z^{-n}, \quad (15)$$

where $Q^{(k)}(z)=1+Q_1^{(k)}(z)=1+\sum_{n=1}^{M}q^{(k)}(n)z^{-n}$.

By (10), (11) and (14), equation (16) is set up to solve the least-squares problem (17) using over-determined equations:

$$U^{(k)}(z)=z^{-L}H(z^{-1})\left(z^{-M}Q^{(k)}(z^{-1})\big/Q^{(k-1)}(z^{-1})\right)$$

$$\Rightarrow U^{(k)}(z)=X^{(k)}(z)z^{-M}(1+Q_1^{(k)}(z^{-1}))$$

$$\Rightarrow U^{(k)}(z)-X^{(k)}(z)z^{-M}=X^{(k)}(z)z^{-(M-1)}Q_1^{(k)}(z^{-1}),$$

(16)

$$\min\|\Delta^{(k)}(z)\|_2=\min\|U^{(k)}(z)\|_2=\min\|C^{(k)}q^{(k)}-d^{(k)}\|_2,$$

(17)

where $q^{(k)}=\left[q^{(k)}(M)\ \cdots\ q^{(k)}(1)\right]^T$,

$$d^{(k)}=-\left[0\cdots0\cdots x^{(k)}(0)\cdots x^{(k)}(L-M-1)\right]^T,$$

$$C^{(k)}=\begin{pmatrix} x^{(k)}(0) & \ldots & 0 \\ & & \vdots \\ \vdots & \ddots & \\ & & x^{(k)}(0) \\ x^{(k)}(L-1) & \cdots & x^{(k)}(L-M) \end{pmatrix}.$$

The algorithm converges after N_T iterations, and we assign $Q(z):=Q^{(N_T)}(z)$. It is proved that for an arbitrary $X^{(k)}(z)$ which minimizes (17) in the least-squares sense, the model is always stable [6]. In summary, a SISO IIR structure can be obtained from $P(z)/Q(z)$ through (11), (12), (16) and (17).

C. *Calculation Issues of Least-Squares*

LS can be classified as a reformulation of Steiglitz-McBride (SM) iteration [10], then an *a priori* error bound for an *M*th-order approximant as stated below,

$$\min_{\deg(P/G)=M}\left(\frac{1}{2\pi}\int_{-\pi}^{-\pi}\left|G(e^{j\omega})-\frac{P^{(i)}(e^{j\omega})}{Q^{(i)}(e^{j\omega})}\right|^{2}d\omega\right)^{1/2}\le\sigma_{N+1}. \quad (18)$$

The error bound can be used to assist in the selection of equalizer order. To determined the error bound, a Hankel matrix is constructed and simplified for a SISO system as,

$$G=\begin{pmatrix} g_1 & g_2 & \cdots & g_n \\ g_2 & & \iddots & 0 \\ \vdots & \iddots & \iddots & \vdots \\ g_n & 0 & \cdots & 0 \end{pmatrix}. \quad (19)$$

It is shown that the singular value of *H* is equivalent as the Hankel singular value of the impulse system [12], and can be calculated efficiently.

In the numerical calculation aspect, LS performs better than other SM-related algorithms since it uses polynomial basis instead of pole-based basis. The numerator can be calculated analytically (polynomial division), which does not involve any numerical-sensitive calculation (e.g. root finding and numerical integration). However, we observe that some ill-conditioned situations in (17) may arise when the algorithm converges; this will cause 'spikes' (i.e., unexpected error) in the result.

D. *MIMO LEast SQuares Construction*

LS algorithm can be further extended to approximate MIMO system response with a common-denominator transfer matrix, where the optimal numerator polynomial can be calculated through (11) and (12) by replacing *G(z)* in (12) by $G_{p,q}$ (z) for input port *p* and output port *q*.

Assuming there are *p* input ports and *q* output ports, the common-used denominator is calculated by the same basis function for all port responses, then the element in each response can be put together into a single over-determined equations:

$$\begin{bmatrix} C^{(k)}{}_{1,1} \\ \vdots \\ C^{(k)}{}_{p,q} \end{bmatrix}\left[q^{(k)}\right]=\begin{bmatrix} d^{(k)}{}_{1,1} \\ \vdots \\ d^{(k)}{}_{q,p} \end{bmatrix}, \quad (23)$$

where $C^{(k)}_{p,q}$ and $d^{(k)}_{p,q}$ are C and d in (17) for input port *p* and output port *q*, respectively. The model order selection criteria for MIMO system can be formed similarly as in the SISO case (19).

In summary, the performance of equalizer design is determined by the following factors:

1) Rate of algorithm convergence,
2) Algorithm computational complexity,
3) Implemented hardware cost and reliability.

The convergence issue will be illustrated in Section 4. For the computational complexity, in a MIMO system, the complexity of M-LESQ is $O\left(N^{(T)}m^2Lpg\right)$ in each iteration, which has a much lower complexity (i.e., less memory storage and computation time) than other approximation methods or model order reduction techniques. Usually, other multi-port system denominator calculation techniques, such as the VF-related ones, require $2\left(\left(p\times q\right)\left(M+1\right)+M\right)$ variables for each iterative pole calculation which have a $O(N^3)$ computation complexity. For the implemented hardware cost and reliability, a common-denominator structure requires less hardware resource (multipliers and adders) for the denominator (feedback path) implementation, which makes it easy and more reliable to implement. This is a significant advantage and meaning of proposing this MIMO Least-SQuares algorithm for the equalizer structure design of MIMO wireless channel.

IV. NUMERICAL EXAMPLES

The extended algorithm M-LESQ is coded in Matlab m-script files and run in the Matlab 7.02 environment on a 4.00GB-RAM 2.4 GHz PC. All 3-ports time signals (the FIR type time domain equalizers examples are generated from [13], total 9 responses) are excited and fitted using M-LESQ with a 35-order model. Time samples are taken for the first 100 points.

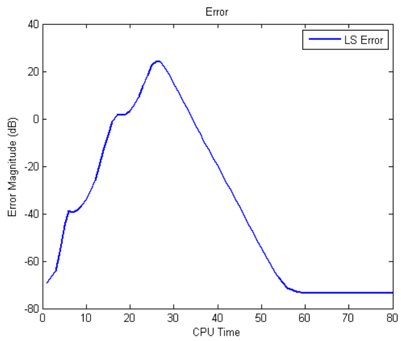

Figure.4 Error for iteration order = 35.

Figure.4 shows the L_2 error during iteration for 35-order model. It shows that M-LESQ converges quickly. In general, M-LESQ converges quickly for a minimal-response, normally within 60 iterations.

As illustrated in Figure.4, M-LESQ generates a 35-order 3*3 model within 1.043 seconds (to achieve convergence), with 0.001 L_2 error which is quite good. Figure.5 and Figure.6 plot the normalized frequency-domain responses and the time-domain responses of the converged approximant, respectively. As we can see in Figure.5, the M-LESQ response shows a quite good and accurate fitting in time domain (with the red line represents the fitting curve with M-LESQ and blue line the original time domain response). In Figure.6, the M-LESQ response (the red line) and original response (the blue line) in the frequency domain are plotted. We can see from both figures that there arises ill-condition situation which cause 'spikes' (unexpected error). Pre-condition methods are investigated to eliminate the ill condition during the calculation and thus remove the 'spikes', and will be reported in the future. Based on above, M-LESQ demonstrates robust and accurate fitting in both the time and frequency domains.

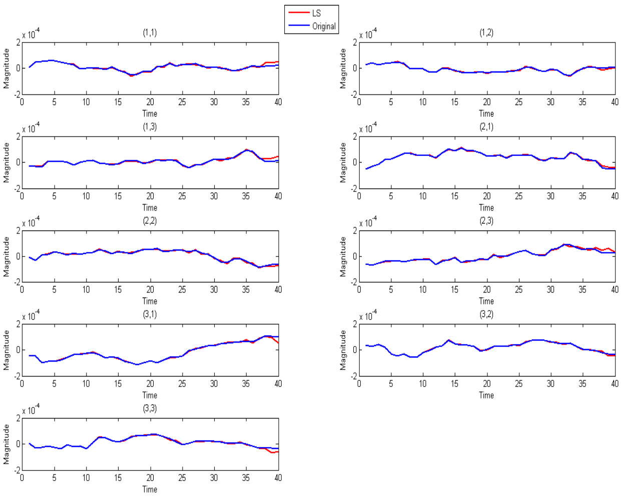

Figure.5 Magnitude response of equalizers example in digital-time domain.

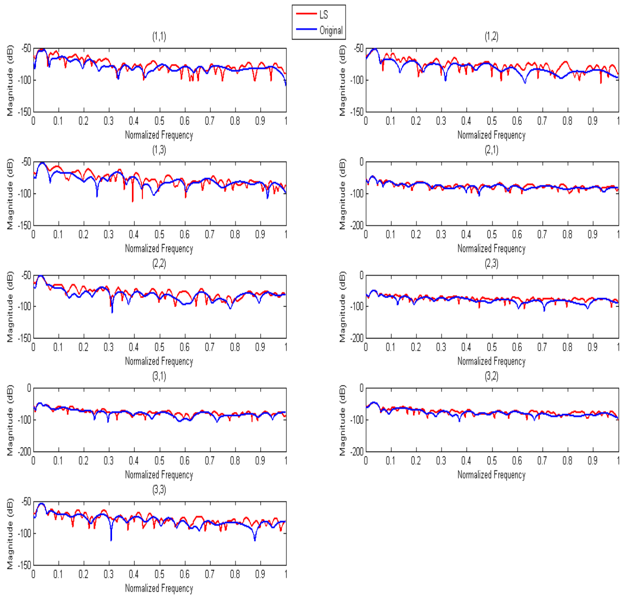

Figure.6 Magnitude response of the equalizers example in normalized frequency domain.

As for hardware saving issue, nine 100-tap FIR type time-domain equalizers are reduced to a 35-order common-denominator IIR structure. The numerator and denominator in each IIR response are of the same order. The total order of these nine FIR type equalizers is $100\times 9 = 900$; while after the fitting, all together the order is $35\times(9+1) = 350$. There is more than 50% save in hardware implementation. Thereby we achieve to demonstrating that M-LESQ is a computational efficient, reasonable accurate and robust algorithm.

V. CONCLUSION

This paper has developed a MIMO approximation technique, which can be efficiently put into the design of hardware-efficient time domain equalizers for MIMO broadband wireless channels. The proposed algorithm, named as M-LESQ (MIMO LEast-SQuares Estimation) is shown to be a simplified Steiglitz-McBride iteration. Application examples have confirmed that the proposed algorithm exhibits an efficient computation with a high accuracy approximation, and generates a hardware saving and robust-implementation structure for equalizer.

ACKNOWLEDGMENT

The authors thank Chi-Un Lei and Ngai Wong from Hong Kong University for their discussion on Least Squares technique and the extention to a MIMO case and Chi-Un Lei's revise on this paper, and the anonymous reviewers for valuable suggested revise to the paper. The funding for this project has been contributed under National HTD 863, National NSF and Shanghai Municipal NSF programs.

REFERENCES

[1] M. Van Bladel and M. Moeneclaey, "Time-domain equalization for multicarrier communication," in Proc. of IEEE Globecomm, Nov. 1995, pp. 167–171.

[2] A. Betser and E. Zeheb, "Reduced order IIR approximation to FIR digital filters," IEEE Trans. Signal Process., vol. 39, no. 11, pp. 2540–2544, Nov. 1991.

[3] B. Beliczynski, I. Kale, and G. D. Cain, "Approximation of FIR by IIR digital filters: An algorithm based on balanced model reduction," IEEE Trans. Signal Process., vol. 40, no. 3, pp. 532–542, Mar. 1992.

[4] H.Brandenstein and R.Unbehauen, "Least-squares approximation of FIR by IIR digital filters," IEEE Trans. Signal Process., vol. 46, no. 1, pp. 21–30, Jan. 1998.

[5] R. W. Aldhaheri, "Design of linear-phase IIR digital filters using singular perturbational model reduction," Proc. Inst. Elect. Eng.—Vision, Image, Signal Process., vol. 147, no. 5, pp. 409–414, Oct. 2000.

[6] H. K. Kwan and A. Jiang, "Recent advances in FIR approximation by IIR digital filters," in Proc. Int. Conf. Comm., Circuits, Systems, Jun. 2006, pp. 185–190.

[7] J. L. Walsh, Interpolation and Approximation by Rational Functions in the Complex Domain Providence.

[8] T. Young and W. Huggins, "'Complementary' signals and orthogonalized exponentials," IRE Trans. On Circuit Theory, vol. 8, no. 4, pp. 362–370, Dec. 1962.

[9] C. U. Lei, H. K. Kwan, Y. Liu, and N.Wong, "Efficient linear macromodeling via least-squares response approximation," in Proc. IEEE Symp. Circuits and Systems, May 2008, pp. 2993–2996.

[10] K. Steiglitza and L. McBride, "A technique for the identification of linear systems," IEEE Trans. Automat. Contr., vol. 10, no. 4, pp. 461–464, Oct. 1965.

[11] G. Gu, "All optimal hankel-norm approximations and their linf error bounds in discrete-time," International Journal of Control, vol.78, no. 6, pp. 408–423, Apr. 2005.

[12] A. Oppenheim and R. Schaffer, Discrete-time Signal Processing. Prentice Hall, 1999.

[13] " UT Austin Multicarrier Equalizer Design Toolbox for Matlab" [Online]. Available: http://users.ece.utexas.edu/~bevans/projects/adsl/dmtteq/dmtteq.html